Drag Coefficient and Velocity of Rise of a Single Collapsing Two-Phase Bubble

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The velocity of rise and the drag of a single vapor bubble collapsing in another immiscible liquid were measured experimentally. During the process of collapse, the dispersed-phase vapor bubble was transformed to a two-phase bubble with condensate accumulating at the rear of the two-phase bubble and vapor at its top. Such a configuration of a two-phase bubble is commonly known as a drobble. Experimental data for the six pairs of liquids covered a range of drobble (two-phase bubble) Reynolds numbers from 0.003 to 3,000. Two regimes of drobble movement were encountered. In the first regime (Re < 100), the drobble maintained its sphericity, and the observed drag was less than the solid-sphere drag predicted by the established solid sphere or Hadamard et al. fluid-sphere drag models. In the second regime, the drobble was deformed and oscillated; the observed drag departed suddenly from predictions of spherical models and increased with increasing Reynolds numbers. The critical Reynolds number covered a range from 100 to 1,000. Empirical models for drobble velocity and drag coefficient are derived.

Introduction

Direct contact heat transfer, with phase change, has the advantage of eliminating metallic heat-transfer surfaces, which are prone to corrosion and fouling, plus smaller required temperature differences, convenient separation of the fluid, and very high heat-transfer coefficients. These advantages can lend themselves to producing economical process systems in instances where the use of a shell and tube heat exchanger would make the process uneconomical. Two such processes of importance are power generation (Boehm et al., 1974, Blair et al., 1976), which utilizes the temperature gradient of geothermal brine, and desalination of sea water (Walker et al., 1967; Sideman, 1966; Othmer, 1962). Some other representative applications of this mode of heat transfer include various schemes associated with cooling of nuclear reactors (Wallis et al., 1975) and refinery operations. Desalination processes provided the impetus for a number of investigators (Sideman and Taitel, 1964; Sideman and Isenberg, 1967; Klipstein, 1963; Simpson et al., 1974; Tochitani et al., 1977a, b; Adams and Pinder, 1972; Raina and Grover, 1988) to investigate the basic mechanism of evaporation of a volatile liquid drop, dispersed in an immiscible liquid. However, for practical applications of this mode of heat transfer, the heat-transfer agent must operate in a closed cycle, and provision must be made to recondense the vapors for reuse. Among earlier investigations on condensation of a bubble in an immiscible liquid medium, are those reported by Isenberg et al. (1970); Moalem and Sideman (1971); Moalem et al. (1973), Sideman and Hirsch (1965), Higeta et al. (1979), and Wanchoo (1991). Their experimental observations reveal that the dynamics of a collapsing two-phase bubble and the mechanism of heat transfer between two immiscible phases are relatively more complex than either that of a drop or a bubble of constant radius. There is no satisfactory theory available that can explain the experimental results and at the same time be consistent with the physical phenomenon.

Sideman and Hirsch (1965) observed a collapsing isopentane bubble in water turn into a two-phase bubble in which the condensate accumulated at the bottom and presumed that heat transfer occurred exclusively across the condensate–continuous-phase interface, occupying the rear part of the

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bubble surface. Isenberg and Sideman (1970) assumed a collapsing two-phase bubble to be covered with a thin film of condensate flowing downward along the surface, into the puddle at the bottom. They further assumed that the velocity of the two-phase bubble would remain constant, and that either the flow of liquid moving past the bubble would be potential flow or the bubble would remain a rigid sphere. The transfer area was assumed to be the front part of the condensate. Isenberg et al. (1970) proposed an approximate analytical solution for bubble collapse based on potential flow and quasi-steady-state assumption. An empirical velocity correction factor was used to approximate the viscous effects. These authors have neglected the resistance of the condensate film and assumed that the bubble surface remains at the instantaneous saturation temperature of the vapor.

Jacobs (1978) observed that, in a two-component system, the condensate film within the bubble, could contribute 30% of the resistance to heat transfer, and concluded that the assumption of potential flow in the condensate film and the external liquid would overpredict the collapse rate. Sideman and Moalem (1982) and Sudhoff et al. (1982) have discussed in detail the heat-transfer characteristics of a drobble collapsing in an immiscible liquid.

The quantitative description of heat transfer in direct contact heat exchangers involving phase change requires knowledge of the transfer area and transfer coefficients in equations, and incorporates the velocity of a single two-phase bubble as a calculation quantity, either directly or indirectly. Clift et al. (1978) have discussed in detail the motion of single-phase fluid particles (drops and bubbles) through an immiscible liquid. The hydrodynamic behavior of such drops and bubbles indicates the lowering of drag in the internal circulation regime and the subsequent transition to a high drag regime. However, there is no conclusive work on momentum transfer involved in collapse of a two-phase bubble in an immiscible liquid available in the literature. Most of the authors have assumed the behavior of a collapsing two-phase bubble to be similar to that of a pure bubble of constant radius, and used the corresponding correlations for drag coefficient, Cd, to predict the bubble velocity. Due to the complex nature of a drobble (representation of a two-phase bubble), involving formation of condensate film, internal circulation, and the varying degree of bubble deformation during the course of collapse, the available correlations for Cd and drobble velocity do not agree with the actual phenomenon. The presence of noncondensables in the dispersed phase vapor adds further complications to the process.

This study is undertaken to observe the drag coefficient and the instantaneous velocity of rise of a two-phase bubble, collapsing in an immiscible quiescent liquid, and to correlate these quantities with some important and easily measurable system variables.

The experimental observations involved the use of cinephotography to record the collapse of *n*-pentane, isopentane, and furan bubble in distilled water or aqueous glycerol solutions (75% wt or 98.3% wt). From this record, the dynamic characteristics of a collapsing two-phase bubble were determined. In order to ensure that the bubbles under study were completely vaporous and free from noncondensables, a modified technique was used to vaporize the dispersed phase liquid. This technique allowed a better control over the bubble release and eliminated the presence of noncondensables, in the dispersed phase fluid.

Six pairs of fluids were used in the study. The properties of the test liquids are shown in Table 1. *n*-Pentane, isopentane (Fluka), and furan (Merck) were of reagent grade. Aqueous glycerol solutions were prepared by mixing distilled water and glycerol (BDH) of reagent grade. Interfacial tensions and kinematic viscosities of aqueous glycerol solutions at a desired temperature were measured by the drop-weight method and Ubbelohde's viscometer, respectively. Other properties were taken from the literature.

Experimental Apparatus and Procedure

Apparatus

Figure 1a shows the experimental apparatus. The main components of the experimental setup are:

- Main column
- Dispersed-phase feeding system
- Photographic equipment

Main Column. The main column consisted of two concentric columns, the inner one being a thin-walled Pyrex glass

Table 1. Properties of the Test Fluids at 20°C

Property	*n*-Pentane	Isopentane	Furan	98.3 Wt. % Aq. Glycerol	75 wt % Aq. Glycerol	Water
Liquid density (kg/m^3)	610	611	923	1,261.08	1,194.85	998
Vapor density (kg/m^3)	2.97	3.06	2.78	—	—	—
Viscosity ($N \cdot s/m^2$)	2.06×10^{-04}	2.105×10^{-04}	7.183×10^{-04}	1.412	0.0355	1.005×10^{-03}
Normal BP (°C)	36.07	27.8	31.35	—	—	—

Interfacial Tensions (mN/m)

System	σ
n-Pentane-water (32°C)	47.83
Isopentane-water (25°C)	47.57
n-Pentane-75 wt. % aq. glycerol (31°C)	32.83
n-Pentane-98.3 wt. % aq. glycerol (24°C)	27.81
Isopentane-98.3 wt. % aq. glycerol (24°C)	27.7
Furan-98.3 wt.% aq. glycerol (24°C)	15.26

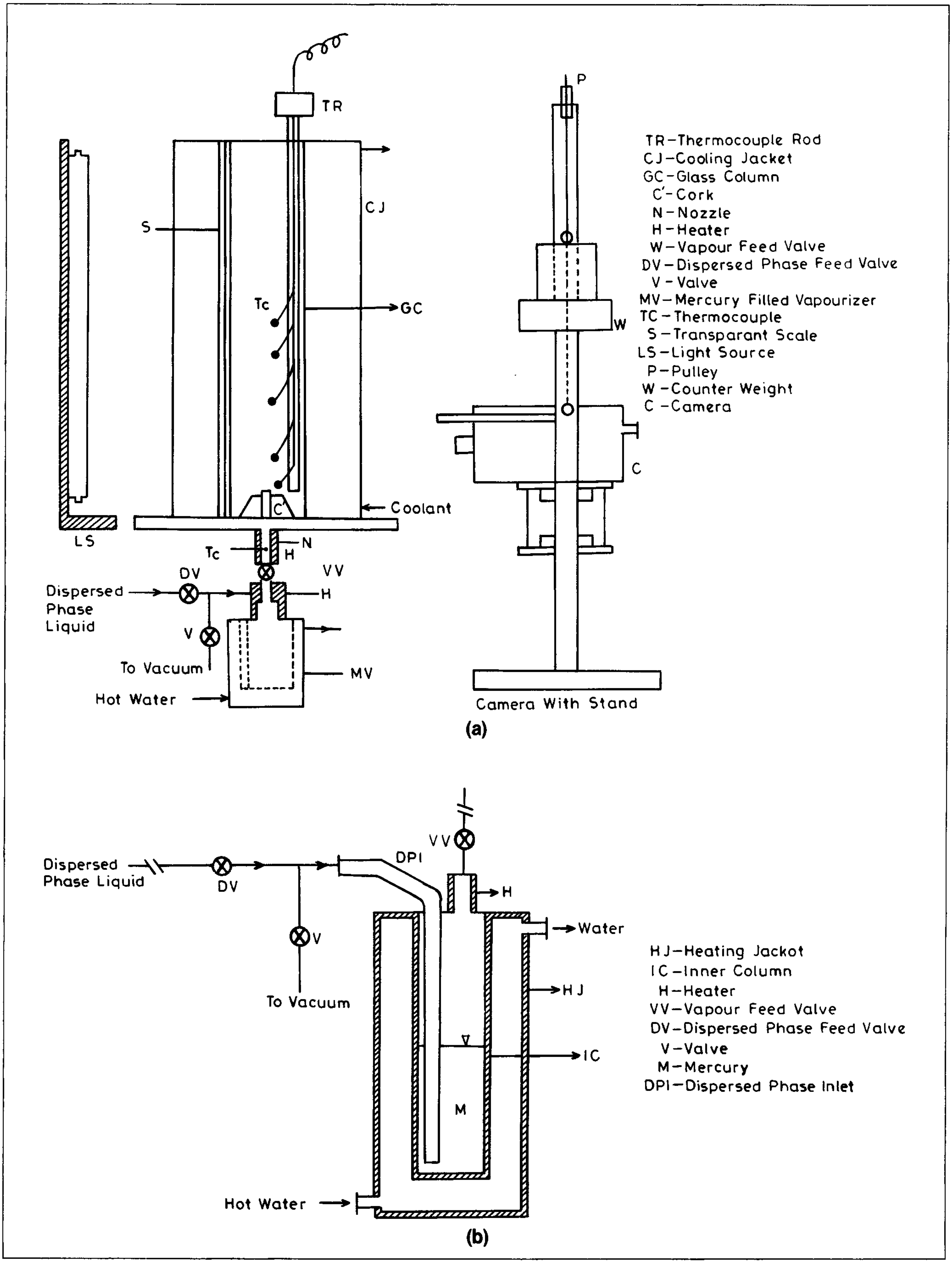

Figure 1. Experimental setup: (a) main column; (b) dispersed-phase vaporizer.

tube of 50 mm ID, and the outer one of transparent Perspex, square in cross section, with dimensions of 125 mm × 125 mm × 800 mm. The square outside column was necessary to minimize the visual distortion caused by the inner cylindrical column. The annular space was used for circulating the coolant (water) from a thermostatically controlled ultracryostat to maintain the temperature of the continuous-phase liquid in the glass column to within ± 0.02 K. The outer column was fixed directly on a 20-mm-thick plate of Perspex, while the inner glass column was closed at the bottom by a rubber cork at the center line and sealed with epoxy resin, except for one hole, which was used for fixing the dispersed-phase nozzle, as shown in Figure 1a. The main purpose of using a rubber cork was to provide an insulating effect around that portion of the nozzle extending into the continuous-phase liquid. The scale of photography was measured by a millimeter scale, placed coplanar with the bubble path. Five copper-constantan (30 gauge) calibrated thermocouples were provided inside the main glass column to measure the temperature of the continuous-phase liquid. The glass column was filled with either distilled water or aqueous glycerol solutions, up to the desired height. The temperature of the continuous-phase liquid was taken as the average of the five thermocouple readings.

Dispersed-phase Feeding System: Dispersed-phase Vaporizer. It consisted of two concentric pipes, the inner one made of stainless steel, surrounded by galvanized iron (GI) pipe (Figure 1b). The inner pipe was half-filled with pure mercury. A dispersed-phase liquid-feed inlet was provided on the side of the inner pipe, extending inside the inner pipe so that the dispersed-phase liquid could be released within the mercury column. The dispersed-phase bubble volume and size were controlled by a needle valve, using brass nozzles of different IDs, fitted at the top of the inner pipe. One copper–constantan thermocouple was embedded inside the nozzle to record the dispersed-phase vapor temperature. In order to avoid any condensation inside the dispersed-phase nozzle, it was heated from the outside by an electrotherm heating element to a superheated temperature of 2–4°C (relative to the saturation temperature of the dispersed-phase liquid).

Dispersed-phase Feeding System: Dispersed-phase Liquid Feeder. Deaerated dispersed-phase liquid was fed into the vaporizer from the top conical glass storage vessel fitted with a reflux condenser located at a height of 2 m from the base of the main column. Deaeration of the dispersed-phase liquid was achieved within an hour by means of an electrotherm heating tape fitted with an energy regulator.

Procedure

The glass column was properly leveled and filled up to a pre-fixed height with either distilled water or aqueous glycerol solution (98.3% wt or 75% wt). The continuous-phase liquid was maintained at the desired temperature by an ultracryostat. Air from the inner pipe of the dispersed-phase vaporizer was evacuated after it was half-filled with pure mercury. The rest of the space inside the vaporizer was filled with continuous-phase liquid by means of very slow filling process, to prevent the formation of air bubbles. The vaporizer was then connected to another constant-temperature bath, the temperature of which was set at a superheat temperature of 2–4°C (relative to the average boiling point of the dispersed-phase liquid). Before a set of experiments with a given dispersed phase, the continuous-phase liquid was always saturated with the dispersed-phase fluid.

After 3 to 4 hours of equilibration, the continuous-phase liquid temperature was steady. Deaerated dispersed-phase liquid was then allowed to pass slowly through the mercury column of the vaporizer, and the formation of the bubble at the nozzle tip was controlled by the needle valve. The dispersed-phase bubble (*n*-pentane or isopentane or furan), thus formed was allowed to move through the continuous-phase column, and its upward motion was recorded by a 16-mm H16RX Paillard Bolex movie camera, using frame speeds of 64, 48, or 24 fps. A backlight arrangement was used for proper exposure. Prior to its motion, the camera was focused on the transparent millimeter plastic scale. Standard spherical glass balls of different sizes were fixed on the plastic scale and were photographed along with the bubble. These standard glass balls helped in getting the actual scale ratio. The feed rate of the dispersed-phase liquid to the vaporizer, along with the position of the feed valve, was so adjusted that a single bubble at a time would enter the main column. The procedure was repeated for various bubble sizes of different dispersed-phase liquids, at different driving forces ($\Delta T = T_d - T_c$).

Each exposed cine film was developed by a reversal process. The developed film was projected onto millimeter-ruled graph paper with a hand-operated movie projector, which allowed the study of individual frames. The magnification was noted by measuring the size of a standard glass ball of known size (fixed on the transparent plastic scale) as it appeared on the graph paper. In the case of spherical or nearly spherical fluid particles, the average of the two axes was taken as its equivalent spherical diameter. For spheroidal fluid particles, the equivalent spherical diameter was calculated from the measured area obtained from the projections of each screen tracing.

Measurements of the film gave equivalent spherical diameter, D, instantaneous two-phase bubble height and time of ascent from the nozzle tip. Instantaneous two-phase bubble velocities and rates of condensation were determined from the film record and the bubble collapse rate, defined by the diameter ratio (D/D_0), where D_0 was the equivalent spherical diameter of the dispersed-phase bubble just before detachment.

Results and Discussion

During experimentation it was observed that:

- The dispersed two-phase bubble would maintain its spherical shape in high-viscous continuous-phase liquid irrespective of initial bubble diameter.
- The amplitude of oscillations was found to decrease with increase in the viscosity of continuous-phase liquid.
- These oscillations were found to be negligible for systems involving the condensation of furan in glycerol, which may be due to a higher dispersed-phase liquid density.
- Throughout the motion of a collapsing two-phase bubble, the vapor phase was always located on top of the liquid phase, and the two phases were always in contact with each other.

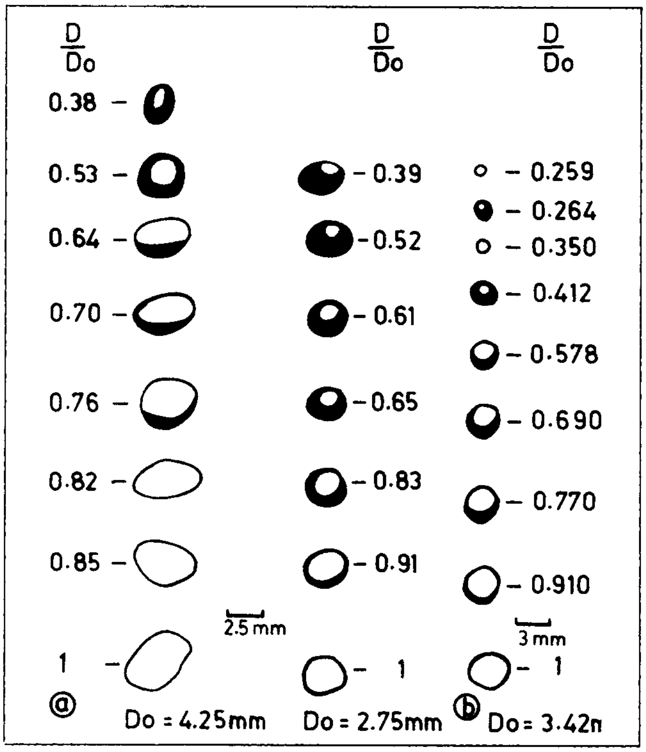

Figure 2. Screen tracings of a collapsing two-phase bubble in an immiscible liquid: (a) *n*-pentane in water; (b) furan in 98.3% glycerol.

- There was a sharp decrease in the velocity of the two-phase bubble toward the last stages of bubble collapse, except when water was used as the continuous phase.
- The two-phase bubble collapsing in water did not maintain its sphericity until the last stages of condensation (degree of condensation >0.8) and always traversed a zigzag path.

Some of the screen tracings of the two-phase bubble (drobble) collapse are shown in Figure 2.

Figure 3 shows representative curves of the experimentally obtained time dependencies of the instantaneous velocities of rise of a isopentane∖*n*-pentane furan bubble collapsing in three different continuous-phase liquids. For the isopentane–water system, the velocity of rise was limited to the narrow range 235–200 mm/s. A wide range of bubble velocities was observed for the *n*-pentane–75% glycerol solution. The velocities varied between 230 and 60 mm/s. In the case of the pentane bubble collapsing in 98.3% glycerol, the velocity of rise varied from 35 mm/s to 12 mm/s.

Development of Correlations

Drag coefficient, Cd

Assuming equilibrium of the drag, gravity, and buoyancy forces, the equation of motion for a collapsing two-phase bubble can be written as

$$u^2/gD = [4/3Cd][1-\rho_{dav}/\rho_c]. \qquad (1)$$

Based on the material balance around a constant-mass two-

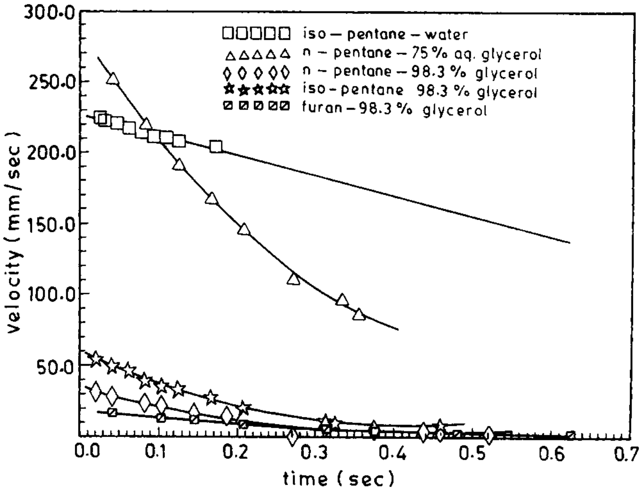

Figure 3. Time dependent velocity of a collapsing two-phase bubble.

phase bubble, the average density of the drobble (Wanchoo and Raina, 1987) can be estimated as

$$\rho_{dav} = \rho_{dv}/(D/D_o)^3. \qquad (2)$$

Combining Eqs. 1 and 2,

$$Cd = [4/3]Ar/Re^2, \qquad (3)$$

where *Ar* is the Archimedes number signifying the hydrodynamics of irregularly shaped bodies with variable density, and is defined as:

$$Ar = [D^3 g\,\rho_c^2/\mu_c^2]\left[1-(D_0/D)^3\,\rho_{dv}/\rho_c\right]. \qquad (4)$$

Based on the observed instantaneous velocity, corresponding drobble drag coefficient, *Cd*, was obtained from Eq. 3 for all the systems studied and plotted against drobble Reynolds number, *Re*, on a log-log scale, as shown in Figure 4. The standard drag coefficient for rigid spheres was calculated from (Beard and Pruppacher, 1969):

$$\begin{array}{ll} Cds = 24/Re & \text{for} Re \le 1 \\ Cds = 24/Re[1+0.102Re^{0.955}] & \text{for } 0.2 < Re \le 2 \\ Cds = 24/Re[1+0.115\,Re^{0.802}] & \text{for } 2 < Re < 21 \\ Cds = 24/Re[1+0.189Re^{0.632}] & \text{for } 21 < Re < 200 \\ Cds = 0.28+6/Re^{0.5}+21/Re & \text{for } 200 < Re < 4{,}000, \quad (5a) \end{array}$$

and is represented by a solid line in Figure 4. The inviscid fluid-sphere model (Eq. 5b) from Hadamard and Rybczynski was also used for predicting *Cd*, and is represented by a dashed line:

$$\begin{array}{ll} Cd = 16/Re & \text{for } Re < 1 \\ Cd = 16/Re[1+Re/8] & \text{for } 1 < Re < 10. \quad (5b) \end{array}$$

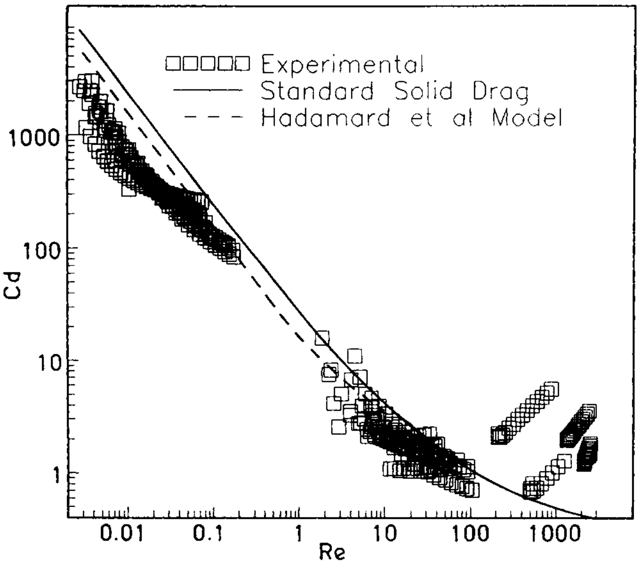

Figure 4. Drag coefficient vs. Reynolds number for collapsing two-phase bubbles in an immiscible liquid.

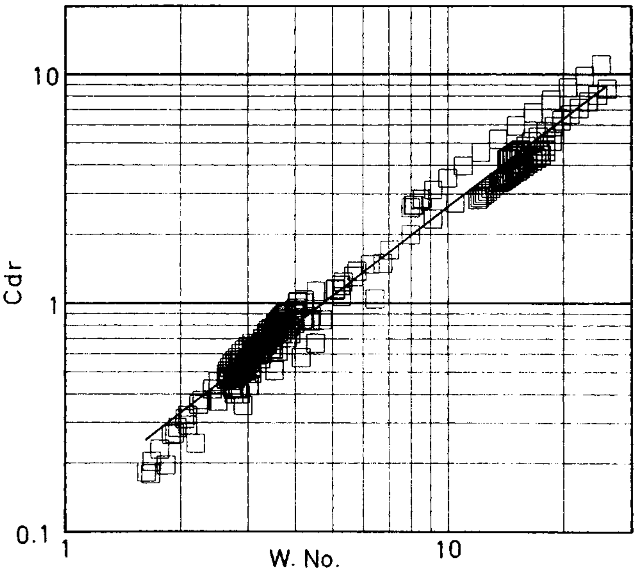

Figure 5. Reduced drag coefficient vs. *W* number for a collapsing two-phase bubble in an immiscible liquid.

The earlier observations of Higeta et al. (1979) on the collapse of an *n*-pentane bubble in glycerol, and Tochitani et al. (1977a, b) on vaporization of pentane or furan drops in glycerol, have shown that the drag coefficients for $0.01 < Re < 10$ are represented by either solid sphere drag ($Re < 1$) or by the Hadamard–Rybczincki fluid-sphere model ($1 < Re < 10$). However, Figure 4 clearly indicates that the collapsing two-phase bubble (drobble) experiences a lower drag compared to that of a solid sphere or a pure fluid sphere of an equivalent size, up to a Reynolds number of 100. The reduction in the drag coefficient of a collapsing two-phase bubble is due to the presence of a mobile interface (codensate film) and a high degree of internal circulation present within the drobble. The onset of internal circulation, even at lower Reynolds numbers ($Re < 0.01$), reduces the skin friction and also the effect of induced drag (or form drag). A higher degree of internal circulation in drops\bubbles\drobbles shifts the point of the hydrodynamic boundary layer separation from the front half of the drobble to its rear. This reduces the size of the wake behind the drobble, that in turn produces a corresponding reduction in induced drag.

With the increase in size, drobble deformation increases rapidly and *Cd* vs. *Re* data pass through a minimum. At still larger bubble sizes, bubble oscillation occurs and there is a sudden rise in drag coefficient. The minimum in *Cd* vs. *Re* corresponds to the critical Reynolds number that could be specific for each system. In the present study, the critical Reynolds number was found to cover a wider range of 100–1,000. Once a drobble attains its critical Reynolds number, its velocity becomes almost independent of its size. This behavior is followed by systems involving low-viscous continuous-phase liquids like water.

In order to arrive at a correlation for *Cd*, the reduced drag coefficient was defined as: $Cdr = Cd/Cds$ and plotted against a new dimensionless group, *W*, defined as

$$W = [Dg^{2/3}\Delta\rho^{2/3}]/[u\mu_c^{1/3}\rho_c^{1/3}].$$

From the normalized data, as shown in Figure 5, an empirical functional relationship between the drobble drag coefficient *Cd* and the *W* number was calculated by least-square regression analysis and is given by

$$Cdr = Cd/Cds = 0.133[W]^{1.3}. \tag{6}$$

From Figure 5, the critical value of *W* corresponding to the onset of oscillations and the introduction of deformation drag (i.e., when $Cdr \geq 1$) is 5. Thus, for $W \geq 5$, the collapsing two-phase bubble shall be moving in an unstable region where deformation drag and drobble oscillations dominate over skin drag. Internal circulation seemed to be absent or had little effect on drag reduction within this region.

Velocity of a collapsing two-phase bubble, u

Various forces that affect the drobble motion through an immiscible liquid are: inertial, viscous, buoyancy, and interfacial tension. In dimensionless analysis, it is a common practice to use force ratios as dimensionless parameters, and since the buoyancy force is responsible for drobble motion in fluids, the geometric mean of the following force ratios could therefore be considered as a measure of interaction: inertia/buoyancy, viscous/buoyancy, interfacial tension /buoyancy. In the form of an equation, this could be expressed as

$$\text{Interaction} = [u\rho_c^{1/3}\mu_c^{1/3}\sigma^{1/3}]/[g\Delta\rho D^{5/3}].$$

The functional dependence of various system variables on drobble velocity can be expressed as (Abbu-El-Hassan, 1968)

$$f(D, \nu_c, \sigma\mu_c^2, g, \rho_c u) = 0,$$

and the Buckingham Π theorem analysis gives the following dimensionless numbers:

$$F = [g^3 D^8 \Delta \rho^3 \rho_c^2 / \mu_c^4 \sigma]^{1/3} = Re/\text{Interaction} \qquad (7)$$

$$V = [u^3 D^2 \rho_c^2 / \mu_c \sigma]^{1/3} = (4Cd/3)/\text{Interaction}. \qquad (8)$$

The observed data on the velocity of the rise of a collapsing two-phase bubble in an immiscible liquid falls on a single line when the drobble V number (velocity) is plotted against the F number (flow) with one point of inflection as shown in Figure 6a. The transition corresponds to the F number of 1,000. Using least-square regression analysis, the functional relationship between V and F (Figure 6b and 6c) can be expressed as

$$V = 0.123\,(F)^{0.725} \qquad \text{for } F \le 1{,}000 \qquad (9)$$

$$V = 0.881\,(F)^{0.308} \qquad \text{for } F > 1{,}000, \qquad (10)$$

with correlation coefficients (R^2) of 0.99 and "sum of squares of residuals" of 10.77 and 0.0488, respectively.

Thus Eqs. 9 and 10 correlate the drobble velocity observed in about 35 (325 data points) runs in systems with the flow number (F) ranging from 10^{-1} to 1.5×10^5 with a mean error of 1.45%, maximum error of 21.5%, and standard deviation of 1.43. The parity plot is given in Figure 7.

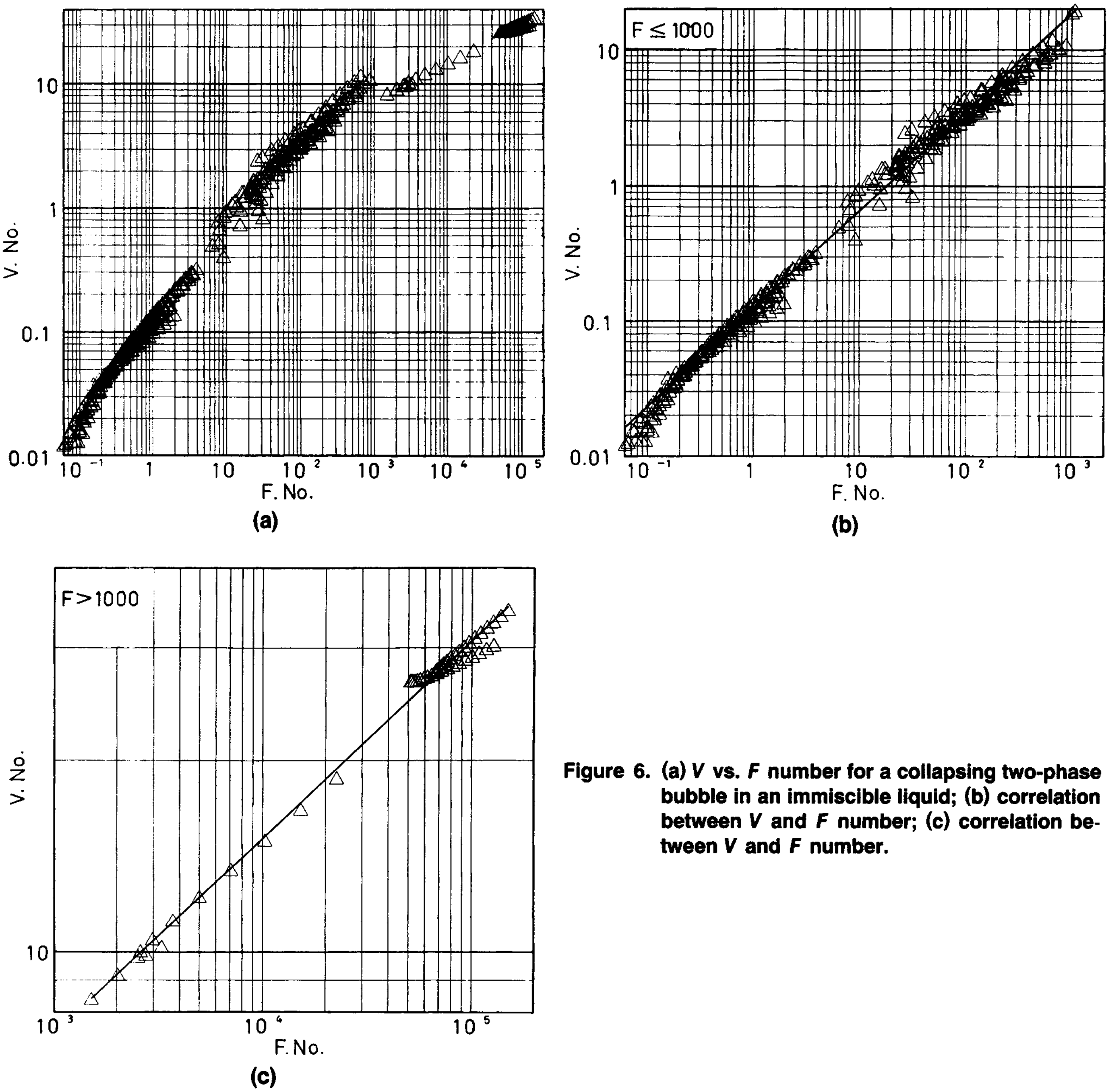

Figure 6. (a) V vs. F number for a collapsing two-phase bubble in an immiscible liquid; (b) correlation between V and F number; (c) correlation between V and F number.

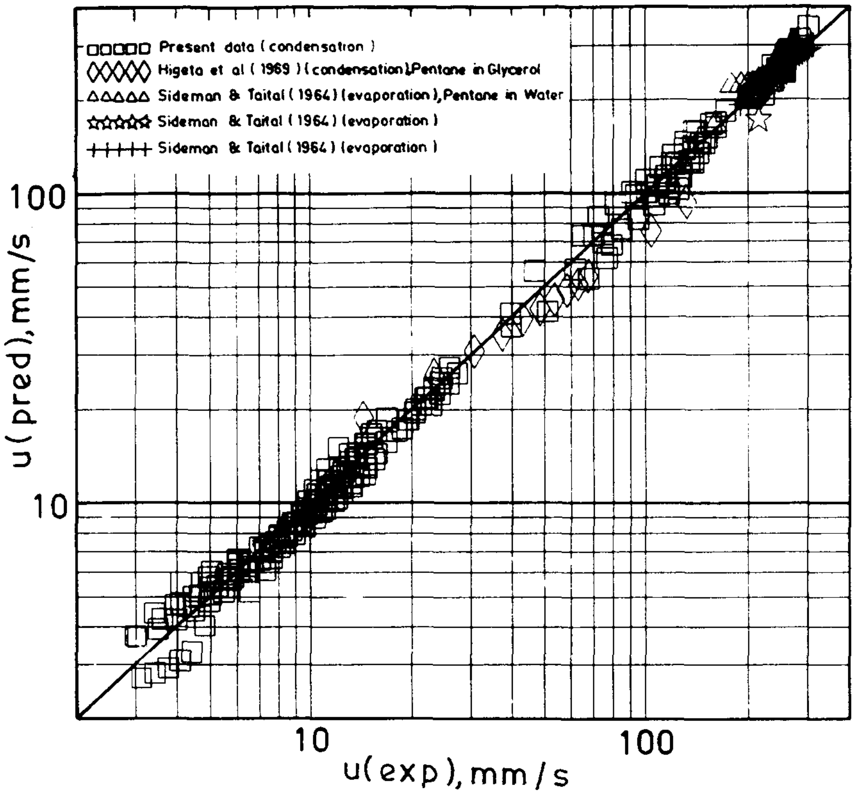

Figure 7. Parity for drobble velocity correlation (Eqs. 9 and 10).

Conclusions

The collapsing two-phase bubble maintains its sphericity in high-viscous continuous-phase immiscible liquids, while it changes its shape and traverses a zigzag path in a low-viscous continuous-phase liquid, like water.

The drobble rises almost at a constant velocity in low-viscosity liquids (viscosity ≤ 1 cp), while a gradual decrease in velocity is observed in other fluids. A sharp decrease in drobble velocity toward the last stages of collapse (condensation ratio > 0.8) may be due to an increase in the average density of the dispersed-phase two-phase bubble.

The collapsing two-phase bubble in an immiscible liquid experiences a lower drag compared to that of solid spheres or pure fluid spheres, until the onset of oscillations characterized by the critical Reynolds number range of 100–1,000. The reduction in the drag coefficient is mainly due to the presence of a mobile interface (condensate film) around the collapsing two-phase bubble and a high degree of internal circulation within the bubble, at least until the last stages of collapse (degree of condensation > 0.8); thereafter, the degree of circulation may decrease due to an increase in the liquid volume within the two-phase bubble. As the drobble motion crosses the critical Reynolds number, deformation drag is more pronounced.

The two-phase bubble drag coefficient is well represented by Eq. 6. The drobble velocity of rise through an immiscible liquid is represented by two dimensionless groups, the *V* number and the *F* number. The correlations represented by Eqs. 9 and 10 predict very well the present experimental data on bubble collapse and those of other investigators involving vaporization of a pentane drop in water (Sideman and Taitel, 1964) and the collapse of a pentane bubble in glycerol (Higeta et al., 1979), as shown in Figure 7.

Notation

g = gravitational acceleration, m^2/s
T_d = saturation temperature of the dispersed-phase bubble, °C
T_c = continuous-phase liquid temperature, °C
u = instantaneous velocity of two-phase bubble, m/s
ρ_c = density of the continuous-phase liquid, kg/m^3
ρ_d = density of dispersed-phase liquid, kg/m^3
ρ_{dv} = density of dispersed-phase vapor, kg/m^3
ρ_{dav} = average density of the two-phase bubble (Eq. 2), kg/m^3
μ_c = viscosity of the continuous-phase liquid, N·s/m^2
ν_c = kinematic viscosity of the continuous-phase liquid, m^2/s
$\Delta\rho$ = density difference, $\rho_{dav} - \rho_c$, kg/m^3
σ = interfacial tension between dispersed- and continuous-phase liquids, N/m

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Manuscript received Sept. 3, 1996, and revision received Apr. 14, 1997.